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Citation: AIP Conference Proceedings **1739**, 020055 (2016); doi: 10.1063/1.4952535
View online: http://dx.doi.org/10.1063/1.4952535
View Table of Contents: http://scitation.aip.org/content/aip/proceeding/aipcp/1739?ver=pdfcov
Published by the AIP Publishing

Numerical Solution of the Problem of Optimizing the Process of Oil Displacement by Steam

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Abstract. The paper is devoted to the problem of optimizing the process of steam stimulation on the oil reservoir by controlling the steam pressure on the injection well to achieve preassigned temperature distribution along the reservoir at a given time of development. The relevance of the study of this problem is related to the need to improve methods of heavy oil development, the proportion of which exceeds the reserves of light oils, and it tends to grow. As a mathematical model of oil displacement by steam, three-phase non-isothermal flow equations is considered. The problem of optimal control is formulated, an algorithm for the numerical solution is proposed. As a reference regime, temperature distribution corresponding to the constant pressure of injected steam is accepted. The solution of the optimization problem shows that choosing the steam pressure on the injection well, one can improve the efficiency of steam-stimulation and reduce the pressure of the injected steam.

INTRODUCTION

Currently, research related to heavy oil extraction is among the most relevant to the oil industry. Technology of oil displacement by steam is widely used as a method for the production of heavy oil reserves, in which increase of oil recovery is achieved by warming up the formation, decreasing the viscosity of the oil, increasing its mobility and others. However, due to the relatively high cost of this technology, research aimed at improving its efficiency is of great practical importance.

We consider a problem of optimizing the process of oil displacement by steam by controlling the steam pressure on the injection well to achieve a predefined temperature distribution along the reservoir at a specified time of development. This problem was studied, for example, in [1] where the technique of solving the optimization problem is based on the adaptive penalty method [2]. To minimize the extended ε-functional, resulting from the application of the penalty method, the gradient descent method was used in [1] and the conjugate set of equations, obtained from the necessary optimality conditions, is numerically solved. The main difficulty of this approach is the derivation of the conjugate problem due to the complexity of the extended ε-functional.

In this paper, we propose a method of solving the considered optimization problem. As a mathematical model of the process of oil displacement by steam, three-phase non-isothermal flow equations, consisting of mass balance equation, Darcy's law and energy equation, is considered [3, 4, 5]. We use so called global formulation of the three-phase non-isothermal flow problem [6, 7] with some simplifications with respect to the physical data. Results of computational experiments are presented for a test problem.

FORMULATION OF THE PROBLEM

We consider a one dimensional problem of oil displacement by steam on the segment $\Omega = [0,\ 1]$, on the left border of which an injection well is placed. We assume that the reservoir is homogeneous and isotropic; capillary, gravitational forces and phase transitions between the phases of water and steam are neglected, water and oil are considered to be incompressible. It is required to approximate the temperature distribution of the reservoir to the reference temperature distribution $\varphi(x)$ at time $t = t_1$ by controlling the injection pressure of steam $p(0,t) = p_*(u(t))$. The function

Innovations Through Mathematical and Statistical Research
AIP Conf. Proc. 1739, 020055-1–020055-6; doi: 10.1063/1.4952535
Published by AIP Publishing. 978-0-7354-1396-2/$30.00

$p_* = p_*(u)$ represents the dependence of saturated vapor pressure on temperature which satisfies the inequality

$$|p_*(u(t))| < \nu_1 |u(t)|, \quad \nu_1 > 0. \tag{1}$$

In other words, it is required to minimize a functional

$$J(u) = \int_0^1 |T(x, t_1, u) - \varphi(x)|^2 \, dx \tag{2}$$

provided that the temperature $T = T(x, t_1, u)$ is determined from the solution of the problem

$$a_1 T_t - k_T T_{xx} = 0, \tag{3}$$

$$a_2 p_t - a_3 p_{xx} = -c_0 T_{xx}, \tag{4}$$

$$\phi \rho_\alpha s_{\alpha,t} - k\lambda_\alpha p_{xx} = 0, \quad \alpha = w, \, o \tag{5}$$

with initial and boundary conditions:

$$p(x, 0) = p_0, \quad T(x, 0) = T_0, \quad s_\alpha(x, 0) = s_{\alpha 0}, \tag{6}$$

$$p(0, t) = p_*(u(t)), \quad p_x(1, t) = 0, \quad T(0, t) = u(t), \quad T_x(1, t) = 0, \tag{7}$$
$$s_\alpha(0, t) = s_{\alpha 1}, \quad s_{\alpha, x}(1, t) = 0, \quad \alpha = w, \, o.$$

The quantities a_i, k_T, c_i, λ_i are considered to be constant. The control u belongs to the set U consisting of functions $u(t) \in H \equiv L_2[0, \, t_1]$ such that

$$M_0 \leq u(t) \leq M_1 \;\; a.e. \, on \;\; [0, \, t_1].$$

The scalar product and norm in H is determined in the form

$$\langle u_1, \, u_2 \rangle_H = \int_0^{t_1} u_1(t) \, u_2(t) \, dt, \quad \|u\|_H^2 = \langle u, \, u \rangle_H.$$

For brevity, we introduce the notation

$$v(x, t) = v(x, t, u) = (p(x, t, u), \, T(x, t, u), \, s_w(x, t, u), \, s_o(x, t, u)).$$

The solution v of the boundary value problem (3)-(7) is uniquely determined for each fixed control u.

METHOD OF SOLVING THE PROBLEM

Let us show that the functional (2) is differentiable. To this end, we take arbitrary controls u, $u + \tilde{u}$. Let $v(x, t, u)$ and $v(x, \, t, \, u + \tilde{u})$ be solutions of the boundary value problems (3)-(7) corresponding to these controls. Let us denote $\tilde{v} = \left(\tilde{p}, \, \tilde{T}, \, \tilde{s}_w, \, \tilde{s}_o\right) = v(x, \, t, \, u + \tilde{u}) - v(x, \, t, \, u)$.

It follows from (3)-(7) that the vector $\left(\tilde{p}, \, \tilde{T}, \, \tilde{s}_w, \, \tilde{s}_o\right)$ is a generalized solution of the problem

$$a_1 \tilde{T}_t - k_T \tilde{T}_{xx} = 0, \tag{8}$$

$$a_2 \tilde{p}_t - a_3 \tilde{p}_{xx} = -b_0 \tilde{T}_{xx}, \tag{9}$$

$$\phi \rho_\alpha \tilde{s}_{\alpha,t} - k\lambda_\alpha \tilde{p}_{xx} = 0, \quad \alpha = w, \, o \tag{10}$$

with initial and boundary conditions:

$$\tilde{p}(x, 0) = 0, \quad \tilde{T}(x, 0) = 0, \quad \tilde{s}_\alpha(x, 0) = 0, \tag{11}$$

$$\tilde{p}(0, t) = p_*(\tilde{u}(t)), \quad \tilde{p}_x(1, t) = 0, \quad \tilde{T}(0, t) = \tilde{u}(t), \quad \tilde{T}_x(1, t) = 0, \tag{12}$$

$$\tilde{s}_\alpha(0,t)=0,\quad \frac{\partial \tilde{s}_\alpha}{\partial x}(1,t)=0.$$

Then the increment of the functional (2) can be expressed as follows

$$\tilde{J}=J(u+\tilde{u})-J(u)=\int_0^1 2\Big(T(x,t_1,u)-\varphi(x)\Big)\tilde{T}(x,t_1)\,dx+\int_0^1\left|\tilde{T}(x,t_1)\right|^2 dx. \tag{13}$$

Note that the first term on the right side of (13) can be expressed in the form [8]

$$\int_0^1 2\Big(T(x,t_1,u)-\varphi(x)\Big)\tilde{T}(x,t_1)\,dx=k_T\int_0^{t_1}\theta_x(0,t)\,\tilde{u}(t)\,dt, \tag{14}$$

where $\theta(x,t,u)=\theta(x,t)$ is determined from the generalized solution of the auxiliary problem

$$a_1\theta_t+k_T\theta_{xx}-c_0\pi_{xx}=0, \tag{15}$$

$$a_2\pi_t+a_3\pi_{xx}+\sum_{\alpha=w,o}k\lambda_\alpha\sigma_{\alpha,xx}=0, \tag{16}$$

$$\phi\rho_\alpha\sigma_{\alpha,t}=0,\quad \alpha=w,\,o \tag{17}$$

with final and boundary conditions

$$\theta(x,t_1)=\frac{2}{a_1}\left(T(x,t_1)-\varphi(x)\right),\quad \pi(x,t_1)=0,\quad \sigma_\alpha(x,t_1)=0, \tag{18}$$

$$\begin{gathered}\theta(0,t)=0,\quad \pi(0,t)=0,\quad \sigma_\alpha(0,t)=0,\\ \theta_x(1,t)=0,\quad \pi_x(1,t)=0,\quad \sigma_{\alpha,x}(1,t)=0.\end{gathered} \tag{19}$$

Substituting (14) into (13), we obtain:

$$\tilde{J}=k_T\int_0^{t_1}\theta_x(0,t)\,\tilde{u}(t)\,dt+\int_0^1\left|\tilde{T}(x,t_1)\right|^2 dx. \tag{20}$$

Let us show that

$$\int_0^1\left|\tilde{T}(x,t_1)\right|^2 dx\le C_1\int_0^{t_1}|\tilde{u}(t)|^2\,dt, \tag{21}$$

where $C_1>0$ does not depend on choice of $u\in H$ and $\tilde{u}\in H$. To derive this estimate, we multiply the equation (8) by $\tilde{T}(x,t)$ and integrate it over the rectangle $Q\equiv[0,1]\times[0,t_1]$:

$$0=\int_0^{t_1}\int_0^1\left(a_1\tilde{T}_t-k_T\tilde{T}_{xx}\right)\tilde{T}\,dx\,dt=$$

$$=\frac{a_1}{2}\int_0^1\left|\tilde{T}(x,t_1)\right|^2 dx+k_T\int_0^{t_1}\tilde{T}_x(0,t)\,\tilde{u}(t)\,dt+k_T\int_0^{t_1}\int_0^1\left|\tilde{T}_x\right|^2 dx\,dt,$$

from which we have

$$\frac{a_1}{2}\int_0^1\left|\tilde{T}(x,t_1)\right|^2 dx+k_T\int_0^{t_1}\tilde{T}_x(0,t)\,\tilde{u}(t)\,dt+k_T\int_0^{t_1}\int_0^1\left|\tilde{T}_x\right|^2 dx\,dt=0.$$

Using ε-inequality $ab\le\frac{\varepsilon a^2}{2}+\frac{b^2}{2\varepsilon}$ valid for any real $a,\,b,\,\varepsilon>0$, we obtain:

$$\frac{a_1}{2}\int_0^1\left|\tilde{T}(x,t_1)\right|^2 dx+k_T\int_0^{t_1}\int_0^1\left|\tilde{T}_x(x,t)\right|^2 dx\,dt\le$$

$$\le\frac{k_T\varepsilon_2}{2}\int_0^{t_1}\left|\tilde{T}_x(0,t)\right|^2 dt+\frac{k_T}{2\varepsilon_2}\int_0^{t_1}|\tilde{u}(t)|^2\,dt. \tag{22}$$

Similarly, multiplying equation (9) by $\tilde{p}(x,t)$ and integrating over the rectangle Q, using the boundary conditions and inequality (1), after obvious transformations, we obtain:

$$\frac{a_2}{2}\int_0^1 |\tilde{p}(x,t_1)|^2\,dx + a_3\int_0^{t_1}\int_0^1 |\tilde{p}_x|^2\,dx\,dt \le \frac{a_3\nu_1}{2\varepsilon_3}\int_0^{t_1}|\tilde{u}(t)|^2\,dt+$$

$$+\frac{b_0\varepsilon_4\nu_1}{2}\int_0^{t_1}|\tilde{u}(t)|^2\,dt + \frac{\varepsilon_5}{2}\int_0^{t_1}\int_0^1 \left|\tilde{T}_x\right|^2 dx\,dt + \frac{1}{2\varepsilon_5}\int_0^{t_1}\int_0^1 |\tilde{p}_x|^2\,dx\,dt. \tag{23}$$

Summing inequalities (22) and (23) and choosing ε_5 such that

$$k_T - \frac{\varepsilon_5}{2} > 0, \quad a_3 - \frac{1}{2\varepsilon_5} > 0,$$

and denoting

$$C_1 = \frac{1}{a_1}\left(\frac{k_T}{\varepsilon_2} + b_0\varepsilon_4\nu_1\right),$$

we obtain the inequality

$$\frac{a_1}{2}\int_0^1 \left|\tilde{T}(x,t_1)\right|^2 dx + \frac{a_2}{2}\int_0^1 |\tilde{p}(x,t_1)|^2\,dx + \left(k_T - \frac{\varepsilon_5}{2}\right)\int_0^{t_1}\int_0^1 \left|\tilde{T}_x(x,t)\right|^2 dx\,dt+$$

$$+\left(a_3 - \frac{1}{2\varepsilon_5}\right)\int_0^{t_1}\int_0^1 |\tilde{p}_x|^2\,dx\,dt \le \frac{2C_1}{a_1}\int_0^{t_1}|\tilde{u}(t)|^2\,dt, \tag{24}$$

which implies the desired estimate (21).

It follows from equation (20) for the increment of the function and estimate (24) that the functional (2) is differentiable and its gradient is as follows

$$J'(u) = k_T\theta_x(0,t,u). \tag{25}$$

We use the gradient projection method for the numerical solution of the problem (2)-(7) which consists in constructing a sequence $\{u_k\}$ according to the rule

$$u_{k+1} = P_U\left(u_k - \alpha_k J'(u_k)\right), \quad k = 0,\ 1,\ \ldots, \quad \alpha_k > 0$$

where $P_U(u)$ is the projection of the point u on the set $U \subset H$, $J(u) \in C^1(U)$. For the problem (2)-(7), it reduces to constructing a sequence by formulas [8]

$$u_{k+1}(t) = \begin{cases} u_k(t) - \alpha_k k_T\theta_x(0,t,u_k), & M_0 \le u_k(t) - \alpha_k k_T\theta_x(0,t,u_k) \le M_1, \\ M_0, & u_k(t) - \alpha_k k_T\theta(0,t,u_k) < M_0, \\ M_1, & u_k(t) - \alpha_k k_T\theta(0,t,u_k) > M_1, \end{cases} \tag{26}$$

where α_k is selected from the condition

$$f_k(\alpha_k) = \inf_{\alpha\ge 0} f_k(\alpha), \quad f_k(\alpha) = J(P_U(u_k - \alpha J'(u_k))).$$

The algorithm for solving the problem (2)-(7) is defined as follows. Let us set an initial control $u = u_0$. Further, direct problem (3)-(7) and conjugate problem (15)-(19) are solved at each iteration. Using the solution of the latter, the gradient of the functional J by (25) is calculated. Finally, the control is determined using (26). The iterative process is performed until the stopping criterion is met.

For the numerical implementation of equations (3)-(5) and (15)-(17), the finite difference method is used. The equations are solved on a uniform grid $\Omega_h = \left\{x_i = i\Delta x \ :\ i = \overline{0,\, N},\ x_0 = 0,\ \ x_N = 1\right\}$.

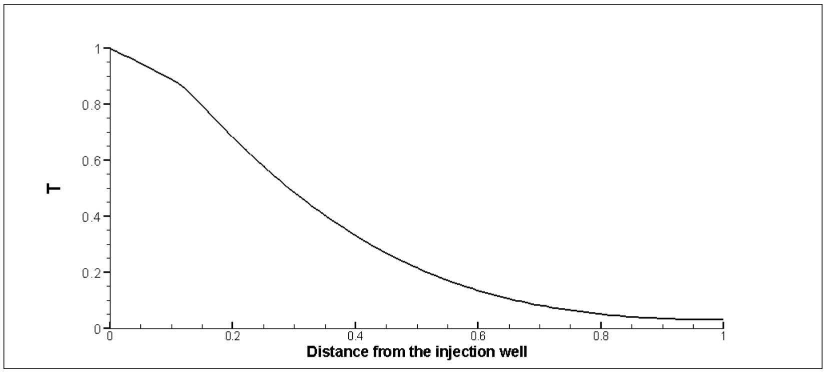

FIGURE 1. Reference temperature distribution

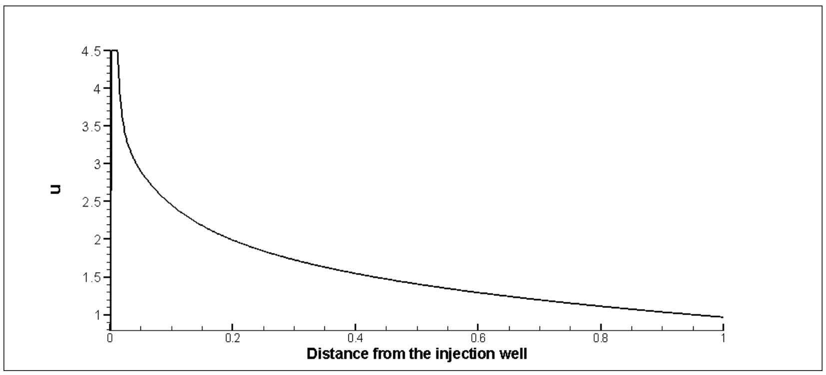

FIGURE 2. Optimal control

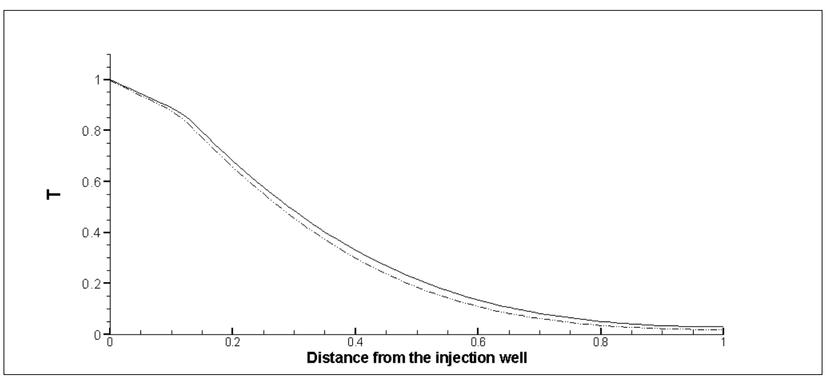

FIGURE 3. Reference temperature (solid line) and a temperature obtained by controlled pressure (dashed line)

NUMERICAL RESULTS AND DISCUSSION

In order to verify the correctness of the algorithm, a test problem was solved. Computational experiments are performed for different values of development time t_1. As the reference temperature, we selected a temperature distribution corresponding to the solution of the problem (3)-(7) with constant pressure of injected steam at time $t_1 = 650\tau$ shown in Fig. 1. Parameter t_1 in the functional (2) is selected to be in range from $t_1 = 450\tau$ to $t_1 = 600\tau$.

In Fig. 2 the control $p_*(u)$ is shown for $t_1 = 450\tau$. Fig. 3 compares the reference temperature with a temperature distribution obtained by controlling the pressure on the 14th iteration. Analyzing the results, one can conclude that to approximate the temperature distribution to the reference temperature, shown in Figure 1, steam injection should be initiated with a certain initial pressure p_1, followed by its decrease. This result agrees with the results obtained in [1].

By varying the value of the parameter t_1, it has been found that reduction of the development period increases the pressure of injected steam on the requirement of a minimum deviation from the reference temperature. Computational experiments show that by controlling the steam pressure in the injection well, the efficiency of the process can be increased by reducing the overall steam consumption. Thus, the results obtained can be used to predict the effectiveness of the operational methods of oil field development. The proposed approach for the numerical solution of the optimal control problem can be applied to similar physical problems.

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