Study of Stability of the Difference Scheme for the Model Problem of the Gaslift Process

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Abstract. The paper studies a model of the gaslift process where the motion in a gas-lift well is described by partial differential equations. The system describing the studied process consists of equations of motion, continuity, equations of thermodynamic state, and hydraulic resistance. A two-layer finite-difference Lax-Vendroff scheme is constructed for the numerical solution of the problem. The stability of the difference scheme for the model problem is investigated using the method of a priori estimates, the order of approximation is investigated, the algorithm for numerical implementation of the gaslift process model is given, and the graphs are presented. The development and investigation of difference schemes for the numerical solution of systems of equations of gas dynamics makes it possible to obtain simultaneously exact and monotonic solutions.

INTRODUCTION

We consider the initial-boundary value problem describing the motion of a liquid in gaslift wells

$$\rho\,(x,\,t)\left(\frac{\partial\vec{v}}{\partial t}+\vec{v}\cdot\nabla\vec{v}\right)+\nabla P(\rho)=-\frac{\lambda_c}{2d_g}\rho\vec{v}|\vec{v}|+\vec{f}, \tag{1}$$

$$\frac{\partial\rho}{\partial t}+\nabla\,(\rho\vec{v})=0, \tag{2}$$

$$P=P\,(\rho)\,, \tag{3}$$

$$\rho|_{t=0}=\rho_0\,(x)\,,\quad \vec{v}|_{t=0}=\vec{v_0}\,(x)\,,\quad \vec{v}|_s=0, \tag{4}$$

in which the unknown functions are the density $\rho(x,\,t)$ and the velocity $\vec{v}(x,\,t)=(v_1(x,\,t),\,...,\,v_n(x,\,t))$ of a viscous compressible fluid filling the bounded domain $\Omega\subset R^n$ with the boundary S. The dimension n is equal to 1, 2 or 3. The velocity $\vec{v_0}(x)$ and the density $\rho_0(x)>0$ are given at the initial time. The coefficient of hydraulic resistance λ_c, the hydraulic channel diameter d_g are some positive constants, and the pressure $P(\rho)$ is a function of a positive argument with the first Lipschitz continuous derivative. The density ρ and the velocity vector $\vec{v}$ are written in the Euler variables $(t,\,x)\in Q=[0,\,T]\cup\Omega$.

The system of equations describing the state of a viscous, compressible fluid, in contrast to the equations (1) also contains the viscous terms on the right-hand side. The solvability of the Cauchy problem on a small time interval for this system was studied in [1] and [2]. A theorem on the local solvability of the initial-boundary value problem for equations of a viscous, compressible fluid was proved in [3]. A local existence theorem for the solution of a one-dimensional initial-boundary value problem for the equations of motion of a viscous perfect polytropic gas in Lagrangian mass coordinates was proved in [4]. In [4] the technique of research of initial-boundary value problems "in the whole" on the time for the system of equations describing the one-dimensional flow of a viscous heat-conducting gas is described.

Various numerical methods for solving the systems of equations for the dynamics of a viscous compressible gas are currently used [5, 6, 7, 8, 9, 10, 11]. However, there is no mathematical proof of their stability and convergence

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to the solution of the differential problem. This is due to the nonlinearity of the equations, and also to the non-evolutionary nature of the system under consideration. For some problems of the dynamics of a viscous barotropic gas, an estimate of the error of difference schemes was obtained in the work of B. G. Kuznetsov and Sh. Smagulov [12, 13]. A new difference scheme for the equations of a one-dimensional viscous heat-conducting gas is proposed and investigated in [14]. Studies of nonlinear difference schemes in the neighborhood of the known solution of specific problems of mathematical physics were carried out in [15, 16].

In [17], the characteristics of various difference schemes for the Euler equations are compared for solutions of a number of model problems of gas dynamics and gas-dynamic processes. A new difference scheme for the nonstationary motion of a viscous barotropic gas in Euler variables is proposed in [18]. Positivity of the density function is ensured by the fact that not the values of the density function themselves are sought, but the natural logarithms of these quantities.

The development and investigation of difference schemes for the numerical solution of systems of equations of gas dynamics makes it possible to obtain simultaneously exact and monotonic solutions.

In the present paper, a two-layer Lax-Vendroff (predictor-corrector type) difference scheme is constructed for solving the problem (1)-(4). A priori estimates for the difference problem are obtained by the method of energy inequalities.

FORMULATION OF THE PROBLEM

Consider first a two-dimensional version of the algorithm:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial z} = 0, \tag{5}$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2}{\partial z} + \frac{\partial p}{\partial z} = -\frac{\lambda_c}{2d_g}\rho u|u| + f, \tag{6}$$

$$\rho(z,0) = \rho_0(z), \quad v(z,0) = v_0(z), \tag{7}$$
$$v(0,t) = v(0,1) = 0.$$

The corresponding difference scheme (5)-(7) is as follows:
- predictor:

$$\frac{\rho_{i+1/2}^{n+1/2} - 0.5\left(\rho_{i+1}^n + \rho_i^n\right)}{\tau/2} + \frac{\rho_{i+1}^n u_{i+1}^n - \rho_i^n u_i^n}{h} = 0, \tag{8}$$

$$\frac{\rho_{i+1/2}^{n+1/2} u_{i+1/2}^{n+1/2} - 0.5\left(\rho_{i+1}^n u_{i+1}^n + \rho_i^n u_i^n\right)}{\tau/2} + \frac{\rho_{i+1}^n \left(u_{i+1}^n\right)^2 - \rho_i^n \left(u_i^n\right)^2}{h} +$$
$$+\frac{p_{i+1}^n - p_i^n}{h} = -\frac{\lambda_c}{2d_g}\rho_i^n u_i^n |u_i^n| + f_i^n; \tag{9}$$

- corrector

$$\frac{\rho_{i+1}^n - \rho_i^n}{\tau} + \frac{\rho_{i+1/2}^{n+1/2} u_{i+1/2}^{n+1/2} - \rho_{i-1/2}^{n+1/2} u_{i-1/2}^{n+1/2}}{h} = 0, \tag{10}$$

$$\frac{\rho_i^{n+1} u_i^{n+1} - \rho_i^n u_i^n}{\tau} + \frac{\rho_{i+1/2}^{n+1/2}\left(u_{i+1/2}^{n+1/2}\right)^2 - \rho_{i-1/2}^{n+1/2}\left(u_{i-1/2}^{n+1/2}\right)^2}{h} +$$
$$+\frac{p_{i+1/2}^{n+1/2} - p_{i-1/2}^{n+1/2}}{h} = -\frac{\lambda_c}{2d_g}\rho_{i+1/2}^{n+1/2} u_i^{n+1/2} |u_i^{n+1/2} + f_i^{n+1/2}, \tag{11}$$

$$\rho_i^0 = \rho_0(z_i), \qquad v_i^0 = v_0(z_i), \qquad i = 1, 2, ..., M-1, \tag{12}$$
$$v_0^n = 0, \quad v_M^n = 0, \quad n = 0, 1, ..., N.$$

Excluding the intermediate values of $\rho_{i+1/2}^{n+1/2}$ and $u_{i+1/2}^{n+1/2}$ from (8), (9), (10), we have

$$\rho_{t,i}^n + (\rho_i^n u_i^n)_{\mathring{x}} - \frac{\tau}{2}\left(\rho_i^n (u_i^n)^2\right)_{\bar{x}x} - \frac{\tau h}{2} p_{\bar{x}x,i}^n - \frac{\tau\lambda_c}{2d_g}\left(\rho_i^n u_i^n |u_i^n|\right)_{\mathring{x}} + \tau f_{\mathring{x},i}^n = 0. \tag{13}$$

STUDY OF STABILITY OF THE DIFFERENCE SCHEME

We first give a procedure for applying energy inequalities for the model transport equation when $u(x, t) = a = const$:

$$y^n_{t,i} + a y^n_{\mathring{x},i} - \frac{a^2\tau}{2} y^n_{\bar{x}x,i} = 0, \tag{14}$$

$$y^n_0 = y^n_M = 0, \quad y^0_i = \rho_0(x_i).$$

Taking into account that $y^n_{\mathring{x},i} = 0.5\left(y^n_{\bar{x},i} + y^n_{x,i}\right)$, rewrite the difference relation (14) in the form

$$y^n_{t,i} + \frac{a}{2}\left(y^n_{\bar{x},i} + y^n_{x,i}\right) - \frac{a^2\tau}{2} y^n_{\bar{x}x,i} = 0. \tag{15}$$

Multiply (15) taken on the n-th layer by $2\tau h y^{n+1}_i$. The resulting equality is summed over i from 1 to $M-1$:

$$2\tau \sum_{i=1}^{M-1} y^n_{t,i} y^{n+1}_i h + a\tau \sum_{i=1}^{M-1} y^n_{\bar{x},i} y^{n+1}_i h + a\tau \sum_{i=1}^{M-1} y^n_{x,i} y^{n+1}_i h - a^2\tau^2 \sum_{i=1}^{M-1} y^n_{\bar{x}x,i} y^{n+1}_i h = 0. \tag{16}$$

Given that

$$2\tau y^n_{t,i} y^{n+1}_i = \left(y^{n+1}_i\right)^2 - (y^n_i)^2 + \tau^2\left(y^n_{t,i}\right)^2 \tag{17}$$

and applying the difference derivative formulas, we have from (16):

$$\|y^{n+1}\|^2 - \|y^n\|^2 + \tau^2\|y^n_t\|^2 - a\tau \sum_{i=0}^{M-1} y^{n+1}_{x,i} y^n_i h + y^{n+1}_M y^n_{M-1} - y^n_0 y^{n+1}_0 - a\tau \sum_{i=1}^{M} y^{n+1}_{\bar{x},i} y^n_i h +$$

$$+ y^{n+1}_M y^n_M - y^n_0 y^{n+1}_1 - a^2\tau^2 \sum_{i=1}^{M} y^{n+1}_{\bar{x},i} y^n_{\bar{x},i} h - y^{n+1}_M y^n_{\bar{x},M} - y^n_{x,0} y^{n+1}_0 = 0. \tag{18}$$

Taking into account boundary conditions (14), we obtain

$$\|y^{n+1}\|^2 - \|y^n\|^2 + \tau^2\|y^n_t\|^2 + a^2\tau^2 \sum_{i=1}^{N} \left(y^n_{\bar{x},i}\right)^2 h =$$

$$= -a^2\tau^3 \sum_{i=1}^{N} y^n_{\bar{x},i} y^n_{\bar{x},t} h + a\tau^2 \left(\sum_{i=0}^{M-1} y^n_{xt,i} y^n_i h + \sum_{i=0}^{M} y^n_{xt} y^n_i h\right). \tag{19}$$

Here the following relations are used:

$$[y^n, y^n_x) + (y^n, y^n_{\bar{x}}] = (y^n, y^n_x) + (y^n, y^n_{\bar{x}}) = (y^n, y^n_x) + y^n_M, y^n_{M-1} - y^n_0, y^n_0 - [y^n_x, y^n) =$$

$$= (y^n, y^n_x) - [y^n_x, y^n) = (y^n, y^n_x) - (y^n, y^n_x) = 0.$$

Estimate the terms on the right-hand side of (19) through the terms of the left-hand side and known quantities as follows:

$$j^n_1 = \left| a^2\tau^3 \sum_{i=1}^{N} y^n_{\bar{x},i} y^n_{\bar{x},t} h \right| = a^2\tau^3 \left(\varepsilon_1^2 \|y^n_{\bar{x}}]|^2 + \frac{1}{4\varepsilon_1^2} \|y^n_{\bar{x},t}]|^2\right).$$

Next, using the well-known inequality [6]

$$\|y^n_{\bar{x}t}]|^2 \le \frac{4}{h^2} \|y^n_t\|^2,$$

we obtain

$$j_1^n \le a^2\tau^3\varepsilon_1^2\|y_{\bar{x}}^n]|^2 + \frac{a^2\tau^2}{\varepsilon_1^2 h^2}\|y_t^n\|^2. \tag{20}$$

We now estimate the second term on the right-hand side of (19) using the inequality [6]:

$$\frac{h^2}{4}\|[y_x\|^2 \le \|y\|^2 \le \frac{l^2}{8}\|[y_x\|^2$$

and ε-inequality:

$$j_2^n = a\tau^2\left|[y_{xt}^n, y^n) + (y_{\bar{x}t}^n, y^n]\right| \le a\tau^2\frac{4}{h^2}\|y_t^n\|\cdot\|y^n\| + a\tau\frac{4}{h^2}\|y_t^n\|\cdot\|y^n\| = \left|[y_{xt}^n, y^n) + (y_{\bar{x}t}^n, y^n]\right|$$

$$= \frac{4a\tau^2}{h}\|y_t^n\|\cdot\|y^n\| \le \frac{a\tau^2 l^2}{2h^2}\|y_t^n\|\cdot\|y_{\bar{x}}^n\| \le \frac{al^2}{2h^2}\left(\left(\tau^{\frac{3}{2}}\|y_t^n\|\right)\cdot\left(\tau^{\frac{1}{2}}\|y_{\bar{x}}^n]|\right)\right) \le \frac{al^2\tau\varepsilon_2^2}{2h^2}\|y_{\bar{x}}^n]|^2 + \frac{al^2\tau^3}{8h^2\varepsilon_2^2}\|y_t^n\|^2. \tag{21}$$

From (19), (20), (21) we have

$$\|y^{n+1}\|^2 - \|y^n\|^2 + \tau^2\left(1 - \frac{a^2\tau}{h^2\varepsilon_1^2} - \frac{al^2\tau}{8h^2\varepsilon_2^2}\right)\|y_t^n\|^2 + \left(a^2\tau^2 - a^2\tau^3\varepsilon_1^2 - \frac{al^2\tau\varepsilon_2^2}{2h^2}\right)\|y_{\bar{x}}^n]|^2 \le 0. \tag{22}$$

Choosing

$$\varepsilon_1^2 = \frac{2a}{h}, \qquad \varepsilon_2^2 = \frac{l^2}{4h},$$

we obtain from (22):

$$\|y^{n+1}\|^2 - \|y^n\|^2 + \tau^2\left(1 - \frac{a\tau}{h}\right)\|y_t^n\|^2 + \left(a^2\tau^2 - \frac{2a^3\tau^3}{h} - \frac{al^4\tau}{8h^3}\right)\|y_{\bar{x}}^n]|^2 \le 0. \tag{23}$$

Imposing the condition on h, τ and a in the form

$$1 - \frac{a\tau}{h} > 0, \tag{24}$$

we obtain the inequality

$$\|y^{n+1}\|^2 - \|y^n\|^2 + a\tau\left(a\tau - \frac{2a^2\tau^2}{h} - \frac{l^4}{8h^3}\right)\|y_{\bar{x}}^n]|^2 \le 0,$$

which allows to estimate $\|y^n\|$ and $\|y_{\bar{x}}^n]|$ under the condition

$$a\tau - \frac{2a^2\tau^2}{h} - \frac{l^4}{8h^3} \ge 0. \tag{25}$$

Indeed, if the coefficient for $\|y_{\bar{x}}^n]|^2$ is nonnegative, therefore

$$\|y^{n+1}\| \le \|y^n\| \le \ldots \le \|y^0\|.$$

Let us write the equation (6) in a non-divergent form taking into account the continuity equation (5) and the density positivity condition ρ:

$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial u^2}{\partial x} + \frac{\partial g}{\partial x} = -\frac{\lambda_c}{2d}|u|\cdot|u| + f, \tag{26}$$

where $g = \ln\rho$.

Then the Lax-Vendroff scheme for the equation (26) has the form

$$\frac{u_{i+\frac{1}{2}}^{n+\frac{1}{2}} - 0.5\left(u_{i+1}^n + u_i^n\right)}{\frac{\tau}{2}} + \frac{1}{2}\left[u_{i+1}^n\frac{u_{i+1}^n - u_i^n}{h} + u_i^n\frac{u_i^n - u_{i-1}^n}{h}\right] +$$

$$+\frac{g_{i+1}^n - g_i^n}{h} = -\frac{\lambda_c}{2d}|u^n_{i+\frac{1}{2}}| \cdot |u^n_{i+\frac{1}{2}}| + f^n_{i+\frac{1}{2}}, \tag{27}$$

$$\frac{u_i^{n+1} - u_i^n}{\tau} + \frac{1}{2}\left[u^{n+\frac{1}{2}}_{i+\frac{1}{2}} \frac{u^{n+\frac{1}{2}}_{i+\frac{1}{2}} - u^{n+\frac{1}{2}}_{i-\frac{1}{2}}}{h} + u^{n+\frac{1}{2}}_{i-\frac{1}{2}} \frac{u^{n+\frac{1}{2}}_{i-\frac{1}{2}} - u^{n+\frac{1}{2}}_{i-\frac{3}{2}}}{h} \right] +$$

$$+\frac{g^{n+\frac{1}{2}}_{i+\frac{1}{2}} - g^{n+\frac{1}{2}}_{i-\frac{1}{2}}}{h} = -\frac{\lambda_c}{2d}|u_i^{n+\frac{1}{2}}| \cdot |u_i^{n+\frac{1}{2}}| + f_i^{n+\frac{1}{2}}. \tag{28}$$

Multiply the equation (27) by $\tau u^{n+\frac{1}{2}}_{i+\frac{1}{2}} h$ and sum along the inner nodes of the grid:

$$2\tau \sum_{i=1}^{M-1} \left(u^{n+\frac{1}{2}}_{i+\frac{1}{2}}\right)^2 h - 2\tau \sum_{i=1}^{M-1} u^{n+\frac{1}{2}}_{i+\frac{1}{2}} u^{n+\frac{1}{2}}_{i+\frac{1}{2}} h + \frac{\tau}{2} \sum_{i=1}^{M-1} u^n_{i+1} u^n_{x,i} u^{n+\frac{1}{2}}_{i+\frac{1}{2}} h +$$

$$+\frac{\tau}{2} \sum_{i=1}^{M-1} u_i^n u^n_{\bar{x},i} u^{n+\frac{1}{2}}_{i+\frac{1}{2}} h + \tau \sum_{i=1}^{M-1} g^n_{x,i} u^{n+\frac{1}{2}}_{i+\frac{1}{2}} h = -\frac{\lambda_c}{2d} \sum_{i=1}^{M-1} |u^n_{i+\frac{1}{2}}| u^{n+\frac{1}{2}}_{i+\frac{1}{2}} h +$$

$$+\tau \sum_{i=1}^{M-1} f_i^{n+\frac{1}{2}} u^{n+\frac{1}{2}}_{i+\frac{1}{2}} h. \tag{29}$$

Estimate scalar products in (29). Let us use the Cauchy inequality and the inequalities from [17] for the term generated by the nonlinear terms:

$$|j_1| \equiv \frac{\tau}{2} \Big| \sum_{i=1}^{M-1} \left(u^n_{i+1} u^n_{x,i} + u^n_i u^n_{\bar{x},i}\right)\left(u^{i+\frac{1}{2}}_{i+\frac{1}{2}} - u^n_{n+\frac{1}{2}}\right) \Big| \leq$$

$$\leq \frac{\tau}{h} \| |u^n|^2 \| \, \|u^{n+\frac{1}{2}} - u_n\| \leq \frac{2\tau}{h} \|u^n\|^{\frac{1}{2}} \|u^n_x\|^{\frac{1}{2}} \|u^{n+\frac{1}{2}} - u^n\| \leq$$

$$\leq \frac{\varepsilon_1 \tau}{h} \|u^{n+\frac{1}{2}} - u^n\|^2 + \frac{2\tau}{\varepsilon_1 h^2} \|u^n\|^2,$$

similarly,

$$|j_2| \equiv \frac{\tau}{2} \Big| \sum_{i=1}^{M-1} \left(u^n_{i+1} u^n_{x,i} + u^n_i u^n_{\bar{x},i}\right) u^n_{n+\frac{1}{2}} \Big| \leq \frac{2\tau}{h} \|u^n\|^{\frac{1}{2}} \|u^n_x\|^{\frac{1}{2}} \|u^n\| \leq \frac{\tau}{h}\left(1 + \frac{2}{h}\right) \|u^n\|^2.$$

Using the difference analogue of the embedding theorem and ε-inequality to the term $j_3 \equiv \frac{\tau \lambda_c}{2d} \sum_{i=1}^{M-1} u^n_{i+\frac{1}{2}} |u^n_{i+\frac{1}{2}}| u^{n+\frac{1}{2}}_{i+\frac{1}{2}} h$, we obtain the inequality:

$$|j_3| \leq \frac{\tau \lambda_c}{2d} \|u^n\|^2 \|u^{n+\frac{1}{2}}\| \leq \frac{\varepsilon_1 \tau \lambda_c}{4d} \|u^n\|^2 + \frac{1}{16\varepsilon_2} \|u^n\|^2 \|u^{n+\frac{1}{2}}_{\bar{x}}]|^2.$$

Using the formula of summation by parts to the last term on the left-hand side of (29), we obtain

$$|j_4| \leq \tau \|u_x^{n+\frac{1}{2}}\| \, \|g\| \leq \frac{\varepsilon_1 \tau}{2} \|u_x^{n+\frac{1}{2}}\|^2 + \frac{\tau}{2\varepsilon_1} \|g\|^2.$$

The term on the right-hand side of the equation (29) is estimated as follows:

$$|j_5| \leq \frac{\tau}{2} \|u_x^{n+\frac{1}{2}}\| \, \|f^n\| \leq \frac{\varepsilon_1 \tau}{16} \|u_x^{n+\frac{1}{2}}\|^2 + \frac{\tau}{8\varepsilon_1} \|f^n\|^2.$$

Substituting these inequalities into (29) and assuming that the inequalities $(\tau h - 2\varepsilon_1)/2h > 0$, $\frac{4}{h^2} - \frac{9\varepsilon_1\tau}{16} - \frac{1}{16\varepsilon_1}\|u^n\|^2 > 0$ hold, we obtain the inequality

$$\|u_x^{n+\frac{1}{2}}\|^2 \leq c_1\|u^n\|^2 + \frac{\tau}{2\varepsilon_1}\|g\|^2 + \frac{\tau}{8\varepsilon_1}\|f^n\|^2. \tag{30}$$

Similarly, multiply the equation (28) by $\tau u_i^{n+1} h$ and sum over the inner nodes of the grid:

$$\|u^{n+1}\|^2 + \tau^2\|u_t^n\|^2 + \frac{4}{h^2}\|u_{\bar{x}}^{n+1}]|^2 + \frac{\tau}{2}\sum_{i=1}^{M-1}\left(u_{i+\frac{1}{2}}^{n+\frac{1}{2}} u_{x,i+\frac{1}{2}}^{n+\frac{1}{2}} + u_{i-\frac{1}{2}}^{n+\frac{1}{2}} u_{\bar{x},i-\frac{1}{2}}^{n+\frac{1}{2}}\right) u_i^n h +$$

$$+\frac{\tau}{2}\sum_{i=1}^{M-1}\left(u_{i+\frac{1}{2}}^{n+\frac{1}{2}} u_{x,i+\frac{1}{2}}^{n+\frac{1}{2}} + u_{i-\frac{1}{2}}^{n+\frac{1}{2}} u_{\bar{x},i-\frac{1}{2}}^{n+\frac{1}{2}}\right) u_{t,i}^n h + \frac{\tau\lambda_c}{2d}|\sum_{i=1}^{M-1} u_i^{n+\frac{1}{2}}|u_i^{n+\frac{1}{2}}|u_i^{n+1} h +$$

$$+\tau\sum_{i=1}^{M-1} u_i^{n+1} g_{x,i-\frac{1}{2}}^{n+\frac{1}{2}} h = 2\|u^n\|^2 + \tau\sum_{i=1}^{M-1} u_i^n f_i^{n+\frac{1}{2}} h. \tag{31}$$

Estimating the scalar products similarly to the equation (29), we obtain a similar estimate

$$\|u^{n+1}\|^2 \leq c_2\|u^n\|^2 + c_3\tau\|g\|^2 + c_3\tau\|f^{n+\frac{1}{2}}\|^2. \tag{32}$$

Adding the inequalities (30) and (32), we obtain

$$\|u^{n+1}\|^2 \leq c_4\|u^n\|^2 + c_5\tau\|g\|^2 + c_6\tau\left(\|f^n\|^2 + \|f^{n+\frac{1}{2}}\|^2\right).$$

Using the inequality obtained, we can show that the chain of inequalities holds:

$$\|u^{n+1}\| \leq \|u^n\| \leq \ldots \leq \|u^0\|.$$

CONCLUSION

Thus, the paper considers the problem of fluid motion in a gas-lift well. A two-layer Lax-Vendroff difference scheme is constructed for the numerical solution of the problem. A priori estimates for the difference problem were obtained by the method of energy inequalities.

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