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An Iterative Method for Solving Nonlinear Navier-Stokes Equations in Complex Domains Taking into Account Boundary Conditions with Uniform Accuracy

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Abstract. In this paper, the Navier-Stokes equations describing the motion of viscous incompressible fluid in a bounded domain is considered. Method of fictitious domains is applied for approximate solution of the problem taking into account boundary conditions with uniform accuracy.

FORMULATION OF THE PROBLEM

In a bounded domain \( \Omega \subset \mathbb{R}^2 \), we consider the initial-boundary value problem for the non-stationary flow of a viscous incompressible fluid. The problem reduces to solving a system of nonlinear Navier-Stokes equations [1]

\[ \frac{\partial v}{\partial t} + (v \cdot \nabla) v = \mu \Delta v - \nabla p + f, \]  
\[ \text{div } v = 0, \]  
\[ v\bigg|_{t=0} = v_0(x), \quad v\bigg|_S = 0. \]  

For simplicity, we assume \( v_0(x) = 0 \). The auxiliary problem corresponding to the method of fictitious domains reduces to solving a system of differential equations in \( D = D_1 \cup \Omega \) [2]

\[ \frac{\partial v^\varepsilon}{\partial t} + (\nu^\varepsilon \cdot \nabla) v^\varepsilon = \text{div} (\mu^\varepsilon \nabla v^\varepsilon) - \nabla p^\varepsilon + f, \]  
\[ \text{div } v^\varepsilon = 0, \]  
\[ v^\varepsilon\bigg|_{t=0} = 0, \quad v^\varepsilon \cdot \tau \bigg|_{S_1} = 0, \quad p^\varepsilon \bigg|_{S_1} = 0, \]  
\[ \mu^\varepsilon = \begin{cases} \mu & \text{in } \Omega, \\ \frac{\mu}{\varepsilon} & \text{in } D_1, \end{cases} \]  
\[ [(\mu^\varepsilon \nabla v^\varepsilon - p^\varepsilon \cdot \delta) n] \bigg|_S = 0, \quad [v^\varepsilon] \bigg|_S = 0. \]  

Here, \( n \) and \( \tau \) are the normal and tangent vector to the boundary \( S_1 \), \( f \) is continued in \( D_1 \) with the preservation of the norm in \( L_2(\Omega) \).

We introduce the set of infinitely differentiable vector-valued functions \( v(x) \) solenoidal in \( D \) with tangential components vanishing on \( S \):

\[ M(D) = \{ v(x) \in C^\infty(D), \quad \text{div } v = 0, \quad v \cdot \tau = 0, \quad x \in S \}, \]
where $\tau$ is the tangent vector to the boundary $S$. The spaces obtained by the closure of $M(D)$ in the norms in $L_2(D)$ and $W_1^2(D)$ are denoted by $V(D)$ and $V_1(D)$, respectively, and their conjugate spaces by $V^*(D)$ and $V_1^*(D)$, and $V(D)$ and $V^*(D)$ are identified.

**Definition 1** A generalized solution of problem (4)-(7) is a function $v^\varepsilon \in L_2(0, T; V_1(D)) \cap L_\infty(0, T; L_2(D))$ satisfying the integral identity

\[- \int_0^T (v^\varepsilon, \Phi) dt - \int_0^T (\Phi \cdot \nabla) dt + \int_0^T \int_{S_1} (v^\varepsilon \cdot \Phi) v^\varepsilon \cdot n ds dt + \frac{\mu}{\varepsilon} \int_0^T \int_{S_1} k(x) (v \cdot \Phi) ds dt + \int_0^T (\mu \varepsilon \nabla v^\varepsilon \cdot \nabla \Phi) dt = \int_0^T (f \cdot \Phi) dt \]

for any $\Phi \in C^1(0, T; V_1(D))$, $\Phi(T) = 0$, $(u, v)_D = \int_D u \cdot v \, dx$. It is assumed that $k(x)$ is a non-negative function.

Let $\omega_1, \omega_2, \ldots, \omega_N$ is an arbitrary basis in $V_1(D)$, and $\tilde{v}_N^\varepsilon$ is an approximate solution of the problem (4)-(7):

$$
\tilde{v}_N^\varepsilon = \sum_{m=1}^N \alpha_{Nm}(t) \omega_m.
$$

(9)

$\alpha_{Nm}(t)$ is found from the system of ordinary differential equations

\[
\frac{d}{dt} \left( \tilde{v}_N^\varepsilon, \omega_j \right)_D + \left( (v^\varepsilon \cdot \nabla) \tilde{v}_N^\varepsilon, \omega_j \right)_D + \frac{\mu}{\varepsilon} \int_{S_1} k(x) (\tilde{v}_N^\varepsilon \cdot \omega_j)_D ds + \left( (\mu \varepsilon \nabla v^\varepsilon \cdot \nabla \omega_j)_D \right) = \left( f, \omega_j \right)_D, \quad j = 1, 2, \ldots, N,
\]

(10)

$$
\tilde{v}_N^\varepsilon(t)|_{t=0} = 0, \quad \alpha_{Nm}(t)|_{t=0} = 0, \quad m = 1, 2, \ldots, N.
$$

(11)

The solvability of (10)-(11) in a small time is known from the general theory of ordinary differential equations [3]. Global solvability follows from a priori estimates of the solution

$$
\max_{0 \leq t \leq T} \left\| \tilde{v}_N^\varepsilon(t) \right\|_{V_1(D)} \leq C < \infty
$$

(12)

which is obtained from system (10).

The following convergence theorem holds [1].

**Theorem 1** Let $f(t) \in L_2(0, T; V_1(D))$, and $\varepsilon$ satisfies the condition

$$
\frac{\mu}{2\varepsilon} - C_0 \int_0^T \left\| f(t) \right\|_{V_1^*(D)} \leq 0.
$$

(13)

Then there exists at least one generalized solution of problem (4)-(7), and the following estimate holds for the solution

\[
\max_{0 \leq t \leq T} \left\| \tilde{v}_N^\varepsilon(t) \right\|_{L_2(D)}^2 + \int_0^T \left\| \nabla \tilde{v}_N^\varepsilon(t) \right\|_{L_2(D)}^2 dt + \frac{1}{\varepsilon} \int_0^T \left\| \nabla \tilde{v}_N^\varepsilon(t) \right\|_{L_2(D)}^2 dt
\]

$$
+ \frac{1}{\varepsilon} \int_0^T \int_{S_1} k(x) \left| \tilde{v}_N^\varepsilon(t) \right|^2 ds dt \leq C \int_0^T \left\| f(t) \right\|_{V_1^*(D)} dt \leq C < \infty.
$$

(14)

In addition, the solution of problem (4)-(7) converges to the solution of problem (1)-(3).

Next, a difference scheme of the second order of approximation is constructed for the problem (4)-(7). For a numerical solution of this difference problem, a special iterative method is constructed that determines approximate solutions on the boundary with uniform accuracy for a limited number of arithmetic operations.

To develop a new numerical implementation algorithm, the idea of the fictitious unknowns method with a two-step iterative process [4] and a method for solving the Poisson difference equation in a square with the right-hand side different from zero only at nodes that are a distance of the order of the grid distance from a given piecewise smooth curve are used [5].
REFERENCES


